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SPECIALIST MATHEMATICS
Teach Yourself Series
Topic 1: Coordinate Geometry

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Coordinate Geometry

In this topic we shall study the graphs of different types of functions-power functions, reciprocal functions, circles, ellipses and hyperbolas. Knowing the graphs of various functions can help us to find different characteristics of the functions- domain, range, shape, max/min values, to name a few.

Graphs of power functions

As it appears in Unit 2 and 3

- The graph of $f(x) = ax + \frac{b}{x}$ is a combination of a linear function with a hyperbola
    - The asymptotes are given by $y = ax$ and $x = 0$
    - The shape is determined by the signs of a and b

- The graph of $f(x) = ax + \frac{b}{x^2}$ is a combination of a linear function with a truncus
    - The asymptotes are given by $y = ax$ and $x = 0$
    - The shape is determined by the signs of a and b

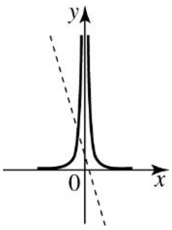
- The graph of $f(x) = ax^2 + \frac{b}{x}$ is a combination of a quadratic function with a hyperbola
    - The asymptotes are given by $y = ax^2$ and $x = 0$
    - The shape is determined by the signs of a and b

- The graph of $f(x) = ax^2 + \frac{b}{x^2}$ is a combination of a quadratic function with a truncus
    - The asymptotes are given by $y = ax^2$ and $x = 0$
    - The shape is determined by the signs of a and b

Eg. Write down the equation of the asymptotes for $y = \frac{8}{x} + 2x + 1$.

Asymptotes are $y = 0$, $x = 0$ (from $y_1 = \frac{8}{x}$) and $y = 2x + 1$.

Eg. The function $y = \frac{4}{x^2} - 3x + 2$ is broken into the functions $y_1 = \frac{4}{x^2}$ and $y_2 = {}^{-}3x + 2$, which appear on the graph below.

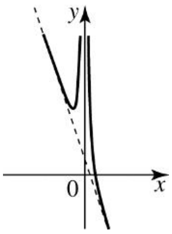


Describe the behaviour of the function $y = \frac{4}{x^2} - 3x + 2$ near the asymptotes. Hence, without any further calculations, sketch the graph of the function.

From y_1, asymptotes are $x = y = 0$.

From y_2, asymptote is also $y = {}^{-}3x + 2$.

As $x \to \pm\infty$, $y \to (-3x + 2)+$, and as $x \to 0$, $y \to \infty$.



Review Questions

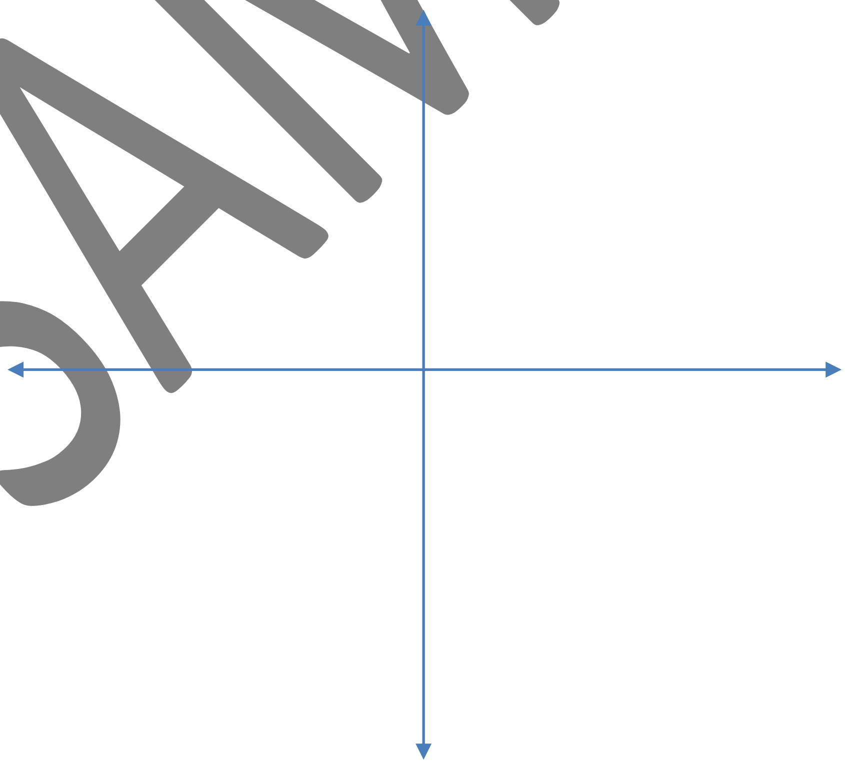
1. Find the asymptotes for each of the following:

a. $y = \frac{1}{x} + 5x$

b. $y = \frac{7}{x^2} + x^2 - 1$

2. Sketch the graph of each function in question **1**

a.



Solutions to Review Questions

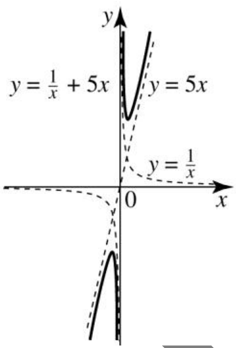
1.

a. Vertical asymptote: $x = 0$ Oblique asymptote: $y = 5x$

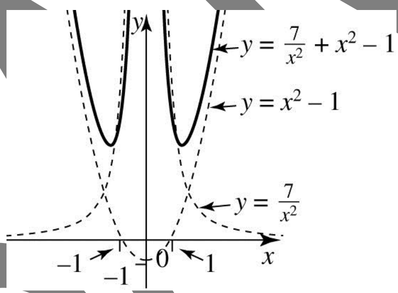
b. Vertical asymptote: $x = 0$ Curved asymptote: $y = x^2 - 1$

2.

a.



b.



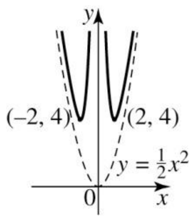
3.

a. no *x*- or *y*-intercepts.

b. Turning points: $\frac{dy}{dx} = 0 = \frac{-16}{x^3} + x$

Therefore, turning points are at $x = 2$, $x = -2$, that is, (2, 4) and (-2, 4).

c.



4.

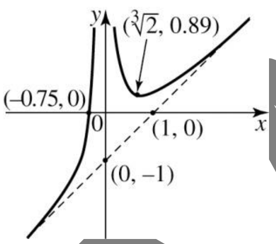
a. x-intercepts: let $y = 0 = \; y = \frac{1}{x^2} + x - 1$.

Therefore, $0 = x^3 - x^2 + 1$ yields x = -0.75

b. Turning points:

$\frac{dy}{dx} = 0 = \frac{-2}{x^3} + 1$, so $x = \sqrt[3]{2}$ and $y \approx 0.89$

c.



5.

a. x-intercept: $2 - x = 0$

$x = 2$

y-intercept: $y = +2 - 0$

$y = 2$

$g(x) = \frac{1}{2-x}$

Vertical asymptote: $x = 2$

Horizontal asymptote: $y = 0$

x-intercept: none since horizontal asymptote is $y = 0$

y-intercept: $y = \frac{1}{2-0} = \frac{1}{2}$

b.